Direct Torque Control of Two-Phase Induction Motors Fed by Two- and Three-leg Inverters

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Abstract— In this paper, direct torque control of two-phase induction motors fed by two- and three-leg inverters is analyzed. Essential equations for controlling electromagnetic torque and stator flux are presented and switching tables are derived. It’s demonstrated that basic method is unable to control the electromagnetic torque properly. The way of changing the switching table for improving torque control is presented. Simulation results, confirm the effectiveness of modified direct torque control of two-phase induction motors.

Keywords—two-phase induction motor, two-leg inverter, three-leg inverter, direct torque control, modified switching table

I. INTRODUCTION

In many household products and also in many low power industrial applications, variable speed performance is needed. The need for variable speed together with concerns about total cost and efficiency has made low-cost drive systems used in low-power applications [1]. Though due to some advantages (such as high torque density, high torque to current ratio and simplicity of control) permanent magnet motor drives have been utilized in many low-power applications and household products [2], single phase induction motors are still pioneers in low-power industrial and domestic applications because of some features such as low cost, simple and robust structure and capability to work when fed by a single phase power supply [3],[4].

Single phase induction motor is an asymmetrical two-phase induction motor including the main and the auxiliary windings that are placed with 90° difference with each other. Three common types of single-phase induction motors are split phase, capacitor-start and capacitor-run motors [5].

In both split-phase and capacitor-start single phase motors, variable speed drive is practically inaccessible as in low speed operation, the centrifugal switch doesn’t disconnect the auxiliary winding and due to the fact that the auxiliary winding is designed to be connected in only starting conditions, it cannot tolerate steady state current and will be damaged.

In capacitor-run single phase motors, the number of turns of the auxiliary winding is more than that of the main winding and so a higher supply voltage is needed for it. The capacitor is in series with the auxiliary winding and is connected in circuit in both starting and steady state conditions [4],[6].

By using an appropriate capacitor, the magnetomotive forces of two motor windings become identical and torque ripple will be suppressed. The appropriate capacitance depends on the conditions of power supply and mechanical load. So, in variable speed application, torque ripple is a constant problem. Moreover, operation of capacitor-run single phase induction motors at low speed can cause some problems such as over temperature, decreased pull-down torque and increased torque ripple [6].

It’s possible to overcome aforementioned problems by utilizing three-leg inverter and using both motor windings. Difficulty of this method is that the voltage of the auxiliary winding should be higher than that of the main winding. In nominal conditions, achieving a higher voltage requires using boost converters or placing a capacitor in series with the auxiliary winding [4]. Using either of these approaches leads to an increased cost and opposes the purpose of cost reduction. Therefore, the best method is speed control of symmetrical two-phase induction motors.

Different inverter topologies have been proposed for two-phase induction motor drives. Common topologies are shown in Fig. 1 [7],[8]. Although the first topology introduces good performance, it opposes the purpose of cost reduction because of using 8 switches. In the second topology, the performance is weak, but due to utilizing only 4 switches it is a low-cost solution. Two-leg inverter is applicable in applications in which two power supplies are available. If two power supplies are not available, utilizing
Nevertheless, when good performance of the motor drive is expected, the use of high-performance methods is inevitable. Field-oriented control of two-phase induction motors has been thoroughly discussed, these methods have not been thoroughly developed for two-phase induction motors [10]. Nevertheless, when good performance of the motor drive is expected, the use of high-performance methods is inevitable. Field-oriented control of two-phase induction motors has been proposed in [11]. Using a position sensor in FOC methods leads to enhanced cost and reduced reliability of the control system. In contrast, DTC method provides good performance of control system without using a speed or position sensor. Utilizing no position sensor has made it a robust and low-cost drive system [12].

In this paper, at first, direct torque control of two-phase induction motor fed by two- and three-leg inverters is proposed based on the method presented in [13]. Then, it is shown that by using these methods, the electromagnetic torque control fails in some regions. Diagnosing these regions is accomplished and modified DTC method for both two- and three-leg inverters is proposed. The proposed method overcomes the problems of the conventional method and is validated by dynamic simulation in Matlab/Simulink.

II. PRINCIPLES OF DIRECT TORQUE CONTROL OF TWO-PHASE INDUCTION MOTORS

For analyzing direct torque control method, on the first place, equations of two-phase induction motor should be studied. It’s assumed that air gap length is constant and magnetic circuit is linear. According to [3], (1)-(4) present voltage-current equations of two-phase induction motors in stationary reference frame.

\[
V_{qs} = r_s i_{qs} + \frac{d}{dt} \lambda_{qs} \tag{1}
\]

\[
V_{ds} = r_s i_{ds} + \frac{d}{dt} \lambda_{ds} \tag{2}
\]

\[
0 = r'_s i'_{QR} + \frac{d}{dt} \lambda'_{QR} - \omega_r \lambda'_{dr} \tag{3}
\]

\[
0 = r'_s i'_{dr} + \frac{d}{dt} \lambda'_{dr} + \omega_r \lambda'_{QR} \tag{4}
\]

where \(r_s\) and \(r'_s\) respectively are the resistance of each phase of stator and rotor, \(V_{qs}\) and \(V_{ds}\) respectively are stator voltages in q and d axes, \(i_{qs}, i_{ds}, i'_{QR}\) and \(i'_d\) respectively are stator current in q axis, stator current in d axis, rotor current in q axis and rotor current in d axis. \(\lambda_{qs}, \lambda_{ds}, \lambda'_{QR}\) and \(\lambda'_{dr}\) respectively are stator flux in q axis, stator flux in d axis, rotor flux in q axis and rotor flux in d axis. d and q axes are in stationary reference frame. Equations (5)-(7) can be used for diagnosing stator flux.

\[
\lambda_{qs} = \int (V_{qs} - r_s i_{qs}) \, dt \tag{5}
\]

\[
\lambda_{ds} = \int (V_{ds} - r_s i_{ds}) \, dt \tag{6}
\]

\[
|\lambda_s| = \sqrt{\lambda_{qs}^2 + \lambda_{ds}^2} \tag{7}
\]

In the model of induction motor, the equations (8) and (9) are established [12].

\[
\lambda_s = L_s i_s + L_m i_r \tag{8}
\]

\[
\lambda_r = L_r i_r + L_m i_s \tag{9}
\]

where \(L_s, L_r\) and \(L_m\) respectively are stator inductance, rotor inductance referred to stator winding and magnetizing inductance. \(i_s\) and \(i_r\) respectively are stator current vector and rotor current vector referred to stator winding. Regarding (1)-(9), the electromagnetic torque of motor is

\[
T_e = P(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) = P \lambda_s x i_s \tag{10}
\]

and (9) give

\[
i_s = \frac{1}{L_s} \lambda_s - \frac{L_m}{L'_s L'_r} \lambda_r \tag{11}
\]

where

\[
L'_s = (1 - \frac{L_m}{L'_s L'_r}) L_s \tag{12}
\]

and (10) and (11) give

\[
T_e = P \frac{L_m}{L'_s L'_r} \lambda_s \times \lambda_r = P \frac{L_m}{L'_s L'_r} |\lambda_s| |\lambda_r| \sin (\theta_s - \theta_r) \tag{13}
\]

where \(\theta_s\) is the stator flux angle and \(\theta_r\) is the rotor flux angle. Electrical time constant of the rotor of induction motors is large (about 0.1 s or more[12]) and so the rotor flux doesn’t change in a short interval and is assumed to be constant. Therefore, a quick response of electromagnetic torque can be achieved by rotating the voltage vector in a way that \(\theta_s\) would increase rapidly. In direct torque control method, at each sampling period, according to desired and actual torque and flux, a voltage vector which reduces the errors of torque and flux is selected.

III. DIRECT TORQUE CONTROL OF TWO-PHASE INDUCTION MOTORS FED BY TWO-LEG INVERTER

Direct torque control of a two-phase induction motor supplied by a voltage source with the middle-point and driven by a two-leg inverter is possible. In this section, first, direct torque control of two-phase induction motors fed by two-leg inverters is proposed based on the method presented in [13]. Then, the switching table will be modified to overcome the torque control problem in basic method.

A. Basic DTC method using two-leg inverter

Fig. 2 shows the available voltage vectors and the way of distinguishing 4 distinct sectors of the stator flux using a method similar to the method presented in [13].

![Figure 2](image-url)
TABLE I. SELECTION OF VOLTAGE VECTORS IN BASIC DTC METHOD IN TWO-PHASE INDUCTION MOTOR FED BY TWO-LEG INVERTER

<table>
<thead>
<tr>
<th>( \Delta \theta )</th>
<th>( \Delta T )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( V_1 )</td>
<td>( V_2 )</td>
<td>( V_3 )</td>
<td>( V_4 )</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>( V_4 )</td>
<td>( V_1 )</td>
<td>( V_2 )</td>
<td>( V_3 )</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>( V_2 )</td>
<td>( V_3 )</td>
<td>( V_4 )</td>
<td>( V_1 )</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>( V_3 )</td>
<td>( V_4 )</td>
<td>( V_1 )</td>
<td>( V_2 )</td>
<td></td>
</tr>
</tbody>
</table>

In each of 4 sectors presented in Fig. 2, at each sampling period, the modulus of stator flux is calculated according to (5)-(7) and the electromagnetic torque is calculated from (10). Calculated values of stator flux and electromagnetic torque are compared with desired values and voltage vector is selected according to the output of hysteresis controllers. The component of voltage vector perpendicular to the flux vector changes the electromagnetic torque and the component of voltage vector along the flux vector changes the modulus of flux vector. Table I presents the switching table of a two-leg inverter supplying a two-phase induction motor based on the approach presented in [13]. Block diagram of direct torque control of a two-phase induction motor is shown in Fig. 3.

When the flux vector is near the border of a sector, the component of voltage vector perpendicular to stator flux becomes too small for reducing the torque error and so the control method fails [10]. So, a method to control the electromagnetic torque in all areas is proposed as follows.

**B. Modified DTC method using two-leg inverter**

As the electromagnetic torque control is the main aim of DTC method, the mentioned problem can be solved by specifying the areas in which the available voltage vectors can’t control both the torque and the flux and then selecting a voltage vector suitable for controlling the torque. In analysis, it’s assumed that the stator flux vector is located in sector 1 and the angle between stator flux vector and \( V_1 \) is \( \alpha \). For increasing both torque and the flux, \( V_1 \) is chosen. At the first moment \( (t_0) \), the electromagnetic torque can be expressed as

\[
T_{e|t=t_0} = p \frac{L_m}{L_{sat}} \beta |\phi_s| |\phi_r| \sin (\alpha - \theta_{e0})
\]  

According to Fig. 4, the component of \( V_1 \) which is perpendicular to flux vector rotates the flux vector. Disregarding ohmic voltage drop, the linear speed of the stator flux is

\[
V_s = V_{1P} = \frac{V_d}{\sqrt{2}} \sin (\alpha)
\]

So, the angular speed of the stator flux is

\[
\omega_s = \frac{V_d}{|\phi_s|} \sqrt{2} \sin (\alpha)
\]

A short time after applying \( V_1 \), the amplitude of stator flux and rotor flux don’t change significantly and can be assumed to be constant. The rotor flux rotates with synchronous speed. So

\[
\theta_{1 | t=t_0 + \Delta t} = \theta_{t_0} + \omega_{syn} \Delta t
\]

The position of the stator flux at the time \( t = t_0 + \Delta t \) can be calculated as

\[
\theta_{1 | t=t_0 + \Delta t} = \alpha + \omega_s \Delta t
\]

At the time \( t = t_0 + \Delta t \), the electromagnetic torque is

\[
T_{e|t=t_0 + \Delta t} = p \frac{L_m}{L_{sat}} \beta |\phi_s| |\phi_r| \sin \left( \frac{V_d}{\sqrt{2} |\phi_s|} \sin \alpha - \omega_s \Delta t + \alpha - \theta_{e0} \right)
\]

The controllability of electromagnetic torque requires the establishment of equation (20)

\[
T_{e|t=t_0 + \Delta t} > T_{e|t=t_0}
\]

According to (9), (20) results in

\[
\alpha \geq \sin^{-1}\left(\frac{\sqrt{2} \omega_s \Delta t}{V_d}\right)
\]

Thus, for the condition in which the stator flux is in the first sector and the angle between stator flux vector and \( V_1 \) is lower than a specified value, the torque control will fail. It can be demonstrated that when the stator flux vector is at the end of any of 4 sectors and both flux and torque should be increased, the same problem occurs. Also it can be demonstrated that when the stator flux vector is at the beginning of any of 4 sectors and flux and torque should be decreased and increased respectively, the mentioned problem also occurs. The minimum angle between the flux vector and border of sectors which guarantees the control of electromagnetic torque is shown by \( \alpha_0 \) that is

\[
\alpha_0 = \sin^{-1}\left(\frac{\sqrt{2} \omega_s |\phi_s|}{V_d}\right)
\]
In direct torque control method, $|\lambda_s|$ and $V_{dc}$ are two specified values. Synchronous frequency also can be found by processing the waveform of voltage, current or flux. So, when the angle between the stator flux and the border of any of sectors is lower than $\alpha_0$, voltage vector capable of reducing the torque error should be applied. For controlling the motor in both clockwise and counterclockwise rotations, the torque can be controlled by using a three-level hysteresis controller. In the proposed method, at each moment the stator flux is located in any of 8 sectors shown in Fig. 5.

According to the mentioned contents, the switching table for a two-phase induction motor supplied by a two-leg inverter is as illustrated in Table II.

The block diagram of modified DTC method for two-leg inverters is shown in Fig. 6. Regarding to Fig. 3, only “$\alpha_0$ diagnosis” block is added that can be a simple circuit for recognition of zero-crossing of motor current or flux. The proposed method makes the torque control possible. Effectiveness of the proposed method is confirmed by simulation results.

### TABLE II. SELECTION OF VOLTAGE VECTORS IN MODIFIED DTC METHOD FOR TWO-PHASE INDUCTION MOTOR FED BY TWO-LEG INVERTER

<table>
<thead>
<tr>
<th>Flux Sector</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>$V_1$</td>
<td>$V_2$</td>
<td>$V_3$</td>
<td>$V_4$</td>
<td>$V_5$</td>
<td>$V_6$</td>
<td>$V_7$</td>
<td>$V_8$</td>
</tr>
<tr>
<td>$d_{-1}$</td>
<td>$V_{-4}$</td>
<td>$V_{-3}$</td>
<td>$V_{-2}$</td>
<td>$V_{-1}$</td>
<td>$V_0$</td>
<td>$V_1$</td>
<td>$V_2$</td>
<td>$V_3$</td>
</tr>
</tbody>
</table>

Figure 5. Proposed 8 sectors of stator flux using two-leg inverter

Figure 6. Block diagram of modified DTC method for two-phase induction motor

IV. DIRECT TORQUE CONTROL OF TWO-PHASE INDUCTION MOTORS FED BY THREE-LEG INVERTER

Using a three-leg inverter for controlling the two-phase induction motor obviates the need of accessing the mid-point of DC-link voltage source. In this section, first, direct torque control of two-phase induction motor based on the method presented in [13] is proposed and then, the switching table will be modified to overcome the problem of torque uncontrollability in basic method.

#### A. Basic DTC method using three-leg inverter

For controlling both flux and electromagnetic torque, the best approach is specifying six distinct sectors for the stator flux. The way of doing this and available voltage vectors are shown in Fig. 7.

In each of six stator flux sectors, at each sampling period the amplitude of stator flux is calculated according to (5), (6) and (7) and the electromagnetic torque is calculated from (10). The calculated values of stator flux and electromagnetic torque are compared with desired values and voltage vector is selected according to the outputs of hysteresis controllers. Table III presents the switching table of a three-leg inverter supplying a two-phase induction motor.

#### B. Modified DTC method using three-leg inverter

The uncontrollability of torque can be solved by specifying the areas in which the available voltage vectors
can’t control both the torque and the flux and selecting a voltage vector suitable for controlling the torque.

It’s assumed that the sector flux vector is located in sector 1 and the angle between stator flux voltage and \( \nu_2 \) is \( \beta \). For increasing both the torque and the flux, \( \nu_2 \) is chosen. Pursuing a process like the process stated in the previous section for two-leg inverters, it can be demonstrated that the establishment of equation (23) guarantees that \( \nu_2 \) increases the electromagnetic torque.

\[
\beta \geq \sin^{-1} \left( \frac{\nu_2 |\lambda_d|}{v_{dc}} \right) \tag{23}
\]

Therefore, for the condition in which the stator flux is in the third sector, when the angle between stator flux vector and \( \nu_3 \) is lower than a specified value, the torque control will fail. It can be shown that when the flux vector is at the end of the sixth sector and both flux and torque should be increased, the same problem exists. Also when the stator flux vector is at the beginning of third and sixth sector and the flux and torque should be decreased and increased respectively, the mentioned problem also exists. The minimum angle between the flux vectors and border of sectors which guarantees the control of electromagnetic torque is shown by \( \beta_0 \) which is

\[
\beta_0 = \sin^{-1} \left( \frac{\nu_3 |\lambda_d|}{v_{dc}} \right) \tag{24}
\]

Similar to what stated about two-phase induction motors supplied by two-leg inverters, in the case of two-phase induction motor supplied by three-leg inverter it’s also possible to control the electromagnetic torque by applying a suitable voltage vector when the stator flux vector is in the first or fourth sector and angle between stator flux vector and borders of the sector is less than \( \beta_0 \). In the suggested method, at each moment the stator flux is located in any of 10 sectors shown in Fig. 8.

According to the stated contents, the switching table of a two-phase induction motor controlled by DTC method and supplied by three-leg inverter, capable of controlling the electromagnetic torque while the rotor is rotating in clockwise and counter clockwise directions is shown in Table IV. The proposed switching table makes the torque control possible. Effectiveness of the proposed method is confirmed by simulation results.

**TABLE IV. SELECTION OF VOLTAGE VECTORS IN MODIFIED DTC METHOD OF TWO-PHASE INDUCTION MOTOR FED BY THREE-LEG INVERTER**

<table>
<thead>
<tr>
<th>Flux Sector</th>
<th>d1</th>
<th>d2</th>
<th>d3</th>
<th>d4</th>
<th>d5</th>
<th>d6</th>
<th>d7</th>
<th>d8</th>
<th>d9</th>
<th>d10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>v2</td>
<td>v2</td>
<td>v2</td>
<td>v2</td>
<td>v2</td>
<td>v2</td>
<td>v2</td>
<td>v2</td>
<td>v2</td>
<td>v2</td>
</tr>
<tr>
<td>0</td>
<td>v2</td>
<td>v2</td>
<td>v2</td>
<td>v2</td>
<td>v2</td>
<td>v2</td>
<td>v2</td>
<td>v2</td>
<td>v2</td>
<td>v2</td>
</tr>
</tbody>
</table>

**V. SIMULATION RESULTS**

Direct torque control drive system of a 2 KW, two-phase induction motor with the parameters mentioned in Table V is simulated by Matlab/Simulink. Both two- and three-leg inverters were utilized to control the motor and in both cases, the DC link voltage, desired torque and desired flux values are 311 V, 8 N.m and 0.84 wb, respectively.

**A. Direct torque control using two-leg inverters**

First, the motor is controlled using two-leg inverter and based on the switching table shown in Table I. The electromagnetic torque and stator flux waveforms are shown in Figures 9.a and 10.a, respectively. Regarding to Fig 9.a, the electromagnetic torque is not controlled properly in some areas, so the modified method is applied to improve the torque control. By processing motor current or flux waveform and determining the rotating field frequency, the value of \( \alpha_0 \) is determined as 270. Then, by controlling the motor based on 8 sectors illustrated in Fig. 5 and using switching table shown in Table II, electromagnetic torque and stator flux waveforms are as shown in Figures 9.b and 10.b, respectively. The proposed method makes torque control possible without noticeable flux destruction.

**TABLE V. PARAMETERS OF TWO-PHASE INDUCTION MOTOR USED IN SIMULATIONS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator Resistance (R_s)</td>
<td>2.6 (\Omega)</td>
</tr>
<tr>
<td>Rotor Resistance (R_r)</td>
<td>1.1 (\Omega)</td>
</tr>
<tr>
<td>Stator Leakage Inductance (L_s)</td>
<td>0.0073 (H)</td>
</tr>
<tr>
<td>Rotor Leakage Inductance (L_r)</td>
<td>0.0073 (H)</td>
</tr>
<tr>
<td>Magnetizing Inductance (L_m)</td>
<td>0.238 (H)</td>
</tr>
</tbody>
</table>

![Figure 9. Electromagnetic torque waveform of two-phase induction motor fed by two-leg inverter using (a) basic method (b) modified method.](image-url)
The motor is also controlled based on the switching table shown in Table III and by using a three-leg inverter. The motor electromagnetic torque and stator flux waveforms are illustrated in Figures 11.a and 12.a, respectively. Regarding to Fig. 11.a, the torque control is failed. So, the proposed method is applied to improve the torque response of drive system. The value of $\beta_0$ is determined as 29° and the motor is controlled again, based on 10 flux sectors mentioned in Fig.10 and using switching table illustrated in Table IV. The electromagnetic torque and stator flux waveforms are shown in Figures 11.b and 12.b, respectively. It can be observed that the proposed method makes torque control possible without noticeable flux control destruction.

VI. CONCLUSION

In this paper, direct torque control of two-phase induction motor fed by two- and three-leg inverters is studied and control algorithms and switching tables are presented. Then, it's shown that in specific areas, the conventional methods are unable to control the electromagnetic torque. The areas in which the electromagnetic torque is uncontrollable are identified and methods based on modifying switching table to improve torque control in all areas are proposed. The simulation results confirm the accuracy of presented formulas for identification of problematic areas and effectiveness of proposed methods to control electromagnetic torque.

REFERENCES


